Self-Insurance, Social Insurance, and the Optimum Quantity of Money

By CHRIS EDMOND*

This paper explores the long-run optimality of deflationary monetary policy in a simple incomplete-markets setting. In the model, a deflationary policy has the standard benefit of raising the rate of return on real balance holdings, which makes consumption-smoothing relatively cheap for those households that want to save. However, for households that want to but cannot borrow (because of the market incompleteness) the lump-sum taxes implied by a deflationary policy result in lower consumption. An optimal policy has to trade off these costs and benefits and can involve using inflation to redistribute resources in favor of would-be borrowers.

I. Environment

Consider an endowment economy populated by an infinite sequence of overlapping generations of two-period-lived households. Each period, a unit mass of new households is born. There is no population growth, and to keep the analysis straightforward, I focus exclusively on stationary states.

Each generation of households is distributed into a number of types. Types are indexed by the characteristic θ with stationary cumulative distribution function $\varphi : \Theta \rightarrow [0, 1]$, where $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ is a strictly positive interval of the real line. A household of type θ has the {youth, old age} endowment profile $\{\theta, y - \theta\}$. The economy's aggregate endowment is not uncertain; it is the constant y each period where $y > \bar{\theta} > \underline{\theta} > 0$, so that all types have strictly positive endowment. Notice that endowments are perfectly *negatively* correlated over the life cycle.

Households may save by holding either government bonds or money. Other types of assets are excluded by assumption, and I also impose the ad hoc borrowing constraint that all households must have nonnegative net assets.

Government policy involves specifying μ, the gross growth rate of the nominal money supply, and α, a constant debt-to-money ratio; $1/(1 + \alpha)$ measures the degree of monetization of public finances. In this paper, I take α as given and consider the welfare and distributional effects of variations in μ. Finally, I do not permit the government to discriminate between households on the basis of either θ type or age; all households are faced with a lump-sum transfer/tax denoted by h. Since there is a unit mass of both young and old, the government's budget constraint is

$$2h + (1 + i)\alpha m + m = \mu(\alpha m + m)$$

which implies the following transfer rule:

$$(1) \qquad h = [(\mu - 1)(1 + \alpha) - \alpha i]\frac{m}{2}$$

$$\equiv \psi(m, i; \mu, \alpha)$$

where i denotes the nominal interest rate paid on government bonds, αm denotes the real quantity of government debt outstanding, and m denotes aggregate real balances—all of which are taken as given by individual households.

Households have identical Cobb-Douglas preferences over consumption when young (c) consumption when old (c') and real balances (z'), with discount factor $\beta \in (0, 1)$ and preference for real balances measured by $\gamma \in [0, 1)$. Although putting money in the utility function eschews all the issues that arise in the pure theory of money, it has the redeeming virtues of ensuring a standard aggregate money demand and that all households will hold at least some money.

* Department of Economics, University of California, Los Angeles, CA 90095. I thank Roger Farmer, Lee Ohanian, and especially Costas Azariadis for their comments, suggestions, and encouragement.

The decision problem of a household of type θ is as follows.

Problem 1:

$$v(\theta, m, i; \mu, \alpha) \equiv \max_{a' \geq 0, z' \geq 0} \left\{ \log(c) + \beta \left[\log(c') + \frac{\gamma}{1-\gamma} \log(z') \right] \right\}$$

subject to

$$c + \mu a' \leq \theta + h$$
$$c' \leq y - \theta + h + (1+i)a' - iz'$$
$$h = \psi(m, i; \mu, \alpha) \qquad c \geq 0 \quad c' \geq 0$$

where a' denotes the net assets and z' denotes the end-of-period real balances of an individual household.

Problem 1 is solved by a pair of decision rules s and ℓ such that for a specification of government policy (μ, α), the total assets held by a type-θ household facing the aggregate state (m, i) are $a' = s(\theta, m, i; \mu, \alpha)$ with corresponding real balances $z' = \ell(\theta, m, i; \mu, \alpha)$.

Definition 1: A *stationary competitive equilibrium* for this economy is a pair of decision rules (s, ℓ) and a pair of nonnegative real numbers (m, i) such that for each policy setting (μ, α) and each household type $\theta \in \Theta$: (i) the decision rules (s, ℓ) solve Problem 1 taking $(\theta, m, i; \mu, \alpha)$ as given, and (ii) markets clear:

$$\int_\Theta \ell(\theta, m, i; \mu, \alpha)\, d\varphi(\theta) = m \tag{2}$$

$$\int_\Theta s(\theta, m, i; \mu, \alpha)\, d\varphi(\theta) = m(\alpha + 1) \tag{3}$$

Notice that not all government policy settings are necessarily consistent with this definition. In particular, if $m > 0$ then the borrowing constraints for all households and market-clearing for bonds can be satisfied if and only if $\alpha \geq -1$. A similar regularity condition is that $\mu \geq \beta$. In what follows, the restrictions $\alpha \in \mathcal{A} \equiv [-1, \infty)$ and $\mu \in \mathcal{M} \equiv [\beta, \infty)$ are enforced.

The deflationary policy $\mu = \beta$ is a version of the rule discussed by Milton Friedman (1969). Friedman's Rule is often optimal in complete-markets economies (see Michael Woodford [1990] and V. V. Chari and Patrick J. Kehoe [1999] for surveys of this literature).

For each policy $(\mu, \alpha) \in \mathcal{M} \times \mathcal{A}$, the equilibrium pair (m, i) simultaneously satisfies the two market-clearing conditions. Thus I look for a pair of functions such that $m = L(\mu, \alpha)$ and $i = I(\mu, \alpha)$. A payoff for a type-θ household is then $v[\theta, L(\mu, \alpha), I(\mu, \alpha); \mu, \alpha]$.

The objective of optimal policy is to maximize the households' *ex ante* expected utility taking as given the equilibrium objects $L(\mu, \alpha)$ and $I(\mu, \alpha)$, which embed the households' best response to (μ, α). This presumes that the government can credibly commit to its choice of policy.

Definition 2: Given $\alpha \in \mathcal{A}$, a monetary policy $\mu^*(\alpha)$ is *optimal* if $w[\mu^*(\alpha); \alpha] \geq w(\mu'; \alpha)$ for all $\mu' \in \mathcal{M}$ where the social welfare function $w(\cdot\,; \alpha) : \mathcal{M} \to \mathbb{R}$ is given by:

$$w(\mu; \alpha) \equiv \int_\Theta v[\theta, L(\mu, \alpha), I(\mu, \alpha); \mu, \alpha]\, d\varphi(\theta). \tag{4}$$

Imagine that, at the beginning of its life, a household does not know its type (although it does know the distribution of types) and that an optimal policy maximizes a household's expected utility with respect to the type distribution φ. Since the distribution of types is exogenous to the economy, this device is formally equivalent to a situation where each household always knows its true type with an optimal policy then defined to be a μ that maximizes "average utility."

II. Analysis

The net asset accumulation rule implied by Problem 1 is given by

$$s(\theta, m, i; \mu, \alpha) = \max\{0, \hat{s}(\theta, m, i; \mu, \alpha)\} \tag{5a}$$

where

$$(5b)\quad \hat{s}(\theta, m, i; \mu, \alpha) = \frac{\beta}{1+\beta-\gamma}\left[\frac{w_1(\theta, h)}{\mu} - \frac{w_2(\theta, h)}{\beta}\left(\frac{1-\gamma}{1+i}\right)\right].$$

In this expression, net incomes in youth and old age are denoted by $w_1(\theta, h) = \theta + h$ and $w_2(\theta, h) = y - \theta + h$. Similarly, the demand function for real balances is given by

$$(6)\quad \ell(\theta, m, i; \mu, \alpha) = \gamma\left[\frac{w_2(\theta, h) + (1+i)s(\theta, m, i; \mu, \alpha)}{i}\right].$$

The level of transfers, h, is defined as in equation (1) above. Notice that all households acquire real balances if $\gamma > 0$ but that not all households necessarily hold net assets.

If a household's borrowing constraint is binding, its demands are given by $c = w_1(\theta, h)$, $c' = (1-\gamma)w_2(\theta, h)$, and $z' = \gamma[w_2(\theta, h)/i]$. For constrained households, both youth and old-age consumption demands are increasing in the level of transfers h and independent of the nominal interest rate i. Real balance demands are increasing in h but decreasing in i. In equilibrium, both h and i will be increasing in the money growth factor μ. Thus, from the point of view of optimal policy, as long as real balances are not given too much weight in the utility function (as long as γ is not too large), constrained households will be relatively favorably disposed toward inflation-financed transfers.

The savings rule given by (5a)–(5b) is weakly increasing in θ. In the terminology of David Gale (1973), a high-θ household will be relatively more "Samuelson" (i.e., will be more inclined to save and hold net assets), while a low-θ household will be relatively more "classical" in nature and is more likely to be constrained.

More formally, by the strict monotonicity of $\hat{s}(\theta, m, i; \mu, \alpha)$ in θ, the equality $\hat{s}(\theta, m, i; \mu, \alpha) = 0$ implicitly determines a cutoff type $\hat{\theta}(m, i; \mu, \alpha)$ such that $s(\theta, m, i; \mu, \alpha) > 0$ for all $\theta > \hat{\theta}(m, i; \mu, \alpha)$, and $s(\theta, m, i; \mu, \alpha) = 0$ for all $\theta \leq \hat{\theta}(m, i; \mu, \alpha)$. The cutoff rule is

$$(7a)\quad \hat{\theta}(m, i; \mu, \alpha) \equiv \begin{cases} \bar{\theta} & \text{if } g(m, i; \mu, \alpha) \geq \bar{\theta} \\ g(m, i; \mu, \alpha) & \text{if } \bar{\theta} \geq g(m, i; \mu, \alpha) \geq \underline{\theta} \\ \underline{\theta} & \text{if } \underline{\theta} \geq g(m, i; \mu, \alpha) \end{cases}$$

where

$$(7b)\quad g(m, i; \mu, \alpha) \equiv \frac{\mu(1-\gamma)}{\mu(1-\gamma)+\beta(1+i)}\, y + \frac{\mu(1-\gamma)-\beta(1+i)}{\mu(1-\gamma)+\beta(1+i)}\, h.$$

Too extreme a policy in one direction or another may cause $\hat{\theta}$ to hit one or the other of the boundary points $\underline{\theta}$ or $\bar{\theta}$. Policy operates along both an intensive and an extensive margin; it can typically change both the portfolio of assets held by households of different types and the fraction of households that are borrowing-constrained.

By substituting equations (5a)–(5b) and (6) into the market-clearing conditions and then using equations (7a)–(7b) to eliminate the max operator, it is possible to derive a simple fixed-point problem in the unknowns (m, i), taking as given the policy (μ, α) and the other structural parameters. Under the assumptions made, this fixed-point problem has a unique solution which can be found using the method of successive approximations (see Edmond [2001] for further details).

III. Self-Insurance and Social Insurance: Examples

In this section I use several numerical examples to illustrate the comparative statics of the model. Suppose that types are normally distributed with mean ϑ and standard deviation ν but that the support of the distribution is truncated to $[\underline{\theta}, \bar{\theta}]$. A convenient choice of units turns out to be $[\underline{\theta}, \bar{\theta}] = [2, 8]$ and $y = 10$.

Since this is a two-period overlapping-generations economy, I set $\beta = 0.5$ to capture

the idea that the length of a period relative to lifespan is long. I assume that the length of a period in calender time is $\Delta = 30$ years. This implies a discount factor in annual terms of $\beta^{1/\Delta} = (0.5)^{1/30} \simeq 0.977$ (i.e., an annual rate of time preference of about 2.3%). With this normalization, I can similarly compute an annual inflation rate as $\pi \equiv \mu^{1/\Delta} - 1$.

In a long-horizon model like this one it seems unrealistic to make γ particularly big. Accordingly, I set $\gamma = 0.025$.

A. *Increasingly Dispersed Type Distributions*

In Table 1, I present the optimal money growth calculations taking as given various settings of α and different amounts of θ-risk. For these calculations, I set $\vartheta = 5$ to ensure that the type distribution is symmetric. Holding α fixed, I vary ν, the standard deviation of the underlying normal, from 0.25 through 2.5. The table reports the annualized optimal inflation rate, nominal and real interest rates, aggregate money demand, and the fraction of households who are net savers.

For fixed α, an increase in θ-risk is associated with a monotonic *fall* in optimal inflation. For example if I set $\alpha = 1$ and the coefficient of variation for θ is low, say, $\mathrm{CV}(\theta) = 0.05$, then the optimal annualized inflation rate is about 0.0214 or just over 2 percent, while if the coefficient of variation is fairly high, say, $\mathrm{CV}(\theta) = 0.31$, the optimal inflation rate is about 1.5 percent. A mean-preserving spread in the type distribution leads to a rise in the fraction of savers.

Panels D–F of Figure 1 show that, while both the level of transfers h and the nominal interest rate i are increasing in the inflation rate, the h schedule is concave in π and the i schedule is convex in π. Any social gains from positive transfers are eventually dominated by the increasing opportunity cost of holding real balances. The trade-offs implied by these two schedules characterize the optimal amount of redistribution.

Notice from equation (1) that if the real interest rate (basically, $1 + i - \mu$) is positive, increasing α means that transfer payments and thus real balance holdings are lower; from equation (7b) this typically implies a reduction in the cutoff point $\hat{\theta}$ and so an increase in the fraction of households that are holders of net assets (as more government debt is supplied to the community). This is illustrated in panels B and E of Figure 1.

Table 1—Increasing the Variance of the Type Distribution for Different Degrees of Monetization

$\pi^*(\alpha)$	$i^*(\alpha)$	$r^*(\alpha)$	$L^*(\alpha)$	Savers
A. $\alpha = -0.5$				
0.0038	0.0090	0.0052	0.4066	2.26×10^{-7}
0.0032	0.0087	0.0055	0.4222	0.0066
−0.0014	0.0070	0.0084	0.5716	0.2912
B. $\alpha = 0.5$				
0.0162	0.0250	0.0086	0.1147	2.37×10^{-5}
0.0158	0.0246	0.0087	0.1168	0.0218
0.0101	0.0210	0.0108	0.1549	0.3263
C. $\alpha = 1.0$				
0.0214	0.0318	0.0102	0.0802	0.0002
0.0214	0.0318	0.0102	0.0803	0.0357
0.0147	0.0272	0.0123	0.1090	0.3478
D. $\alpha = 2.0$				
0.0300	0.0435	0.0131	0.0481	0.0029
0.0293	0.0429	0.0132	0.0494	0.0872
0.0237	0.0391	0.0150	0.0632	0.3880

Notes: Under each value of α, the three rows present results for [$\nu = 0.25$, $\mathrm{CV}(\theta) = 0.050$], [$\nu = 0.50$, $\mathrm{CV}(\theta) = 0.100$], and [$\nu = 2.50$, $\mathrm{CV}(\theta) = 0.314$], respectively. Definitions: α, debt-to-money ratio; ν, standard deviation of the underlying normal distribution; $\mathrm{CV}(\theta)$, coefficient of variation of the type distribution; $\pi^*(\alpha)$, annual inflation rate at the optimum; $i^*(\alpha)$, annual nominal interest rate at the optimum; $r^*(\alpha)$, annual real interest rate at the optimum; $L^*(\alpha)$, aggregate demand for real balances at the optimum; Savers, fraction of households that save at the optimum.

B. *Skewed Type Distributions*

I create type distributions with varying amounts of positive or negative skew by shifting the mean of the underlying distribution, ϑ, relative to the truncation points $[\underline{\theta}, \bar{\theta}] = [2, 8]$. I set the standard deviation of the underlying normal distribution to $\nu = 2.5$ and then vary ϑ from $\vartheta = \underline{\theta} = 2$, which produces a positively skewed distribution, to $\vartheta = \bar{\theta} = 8$, which produces a negatively skewed distribution. In Table 2, I report the annualized optimal inflation rate, nominal and real interest rates, aggregate money demand, and the fraction of households who are net savers.

For a given value of α, the more skewed is the distribution toward low-θ types (the more positive skew), the higher is the optimal rate of

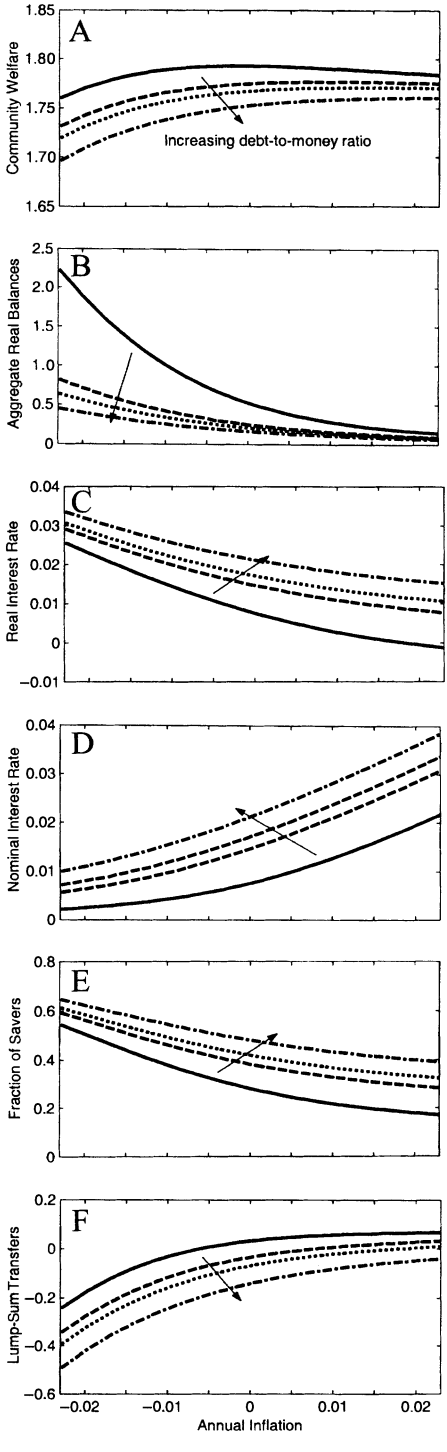

FIGURE 1. EFFECTS OF INCREASING THE STEADY-STATE DEBT-TO-MONEY RATIO

TABLE 2—SHIFTING THE SKEW OF THE TYPE DISTRIBUTION FOR DIFFERENT DEGREES OF MONETIZATION

$\pi^*(\alpha)$	$i^*(\alpha)$	$r^*(\alpha)$	$L^*(\alpha)$	Savers
A. $\alpha = -0.5$				
0.0061	0.0115	0.0053	0.3802	0.0733
−0.0014	0.0070	0.0084	0.5716	0.2912
−0.0143	0.0031	0.0176	1.3119	0.7221
B. $\alpha = 0.5$				
0.0225	0.0320	0.0093	0.0985	0.0974
0.0101	0.0210	0.0108	0.1549	0.3263
−0.0095	0.0094	0.0191	0.4019	0.7423
C. $\alpha = 1.0$				
0.0283	0.0399	0.0113	0.0694	0.1118
0.0147	0.0272	0.0123	0.1090	0.3478
−0.0054	0.0134	0.0190	0.2641	0.7398
D. $\alpha = 2.0$				
0.0385	0.0539	0.0148	0.0405	0.1397
0.0237	0.0391	0.0150	0.0632	0.3880
0.0007	0.0209	0.0202	0.1538	0.7562

Notes: Under each value of α, the three rows present results for [$\vartheta = 2$, skew(θ) = 0.7187], [$\vartheta = 5$, skew(θ) = 0.0000], and [$\vartheta = 8$, skew(θ) = −0.7187], respectively. Definitions: α, debt-to-money ratio; ϑ, mean of the underlying normal distribution; skew(θ), skewness coefficient of the type distribution; $\pi^*(\alpha)$, annual inflation rate at the optimum; $i^*(\alpha)$, annual nominal interest rate at the optimum; $r^*(\alpha)$, annual real interest rate at the optimum; $L^*(\alpha)$, aggregate demand for real balances at the optimum; Savers, fraction of households that save at the optimum.

money creation and inflation, the higher is the nominal interest rate, and the lower is the real interest rate and the fraction of households that are net savers. For example if $\alpha = 1$ and the economy has $\vartheta = 2$ such that the type distribution is biased toward low θ, skew(θ) > 0, then the optimal inflation rate is 0.0283, nearly 3 percent. Alternatively, if $\vartheta = 8$ such that the type distribution is biased toward high θ, skew(θ) < 0, then the optimal inflation rate is −0.0054, a mildly deflationary policy.

Although a deflationary policy can be optimal if the type distribution has enough skew in favor of high θ, the optimal nominal interest rate always remains positive.

C. *Equilibrium Distributions of Consumption and Assets*

Figure 2 illustrates the equilibrium distributions of young consumption, old-age consumption, net

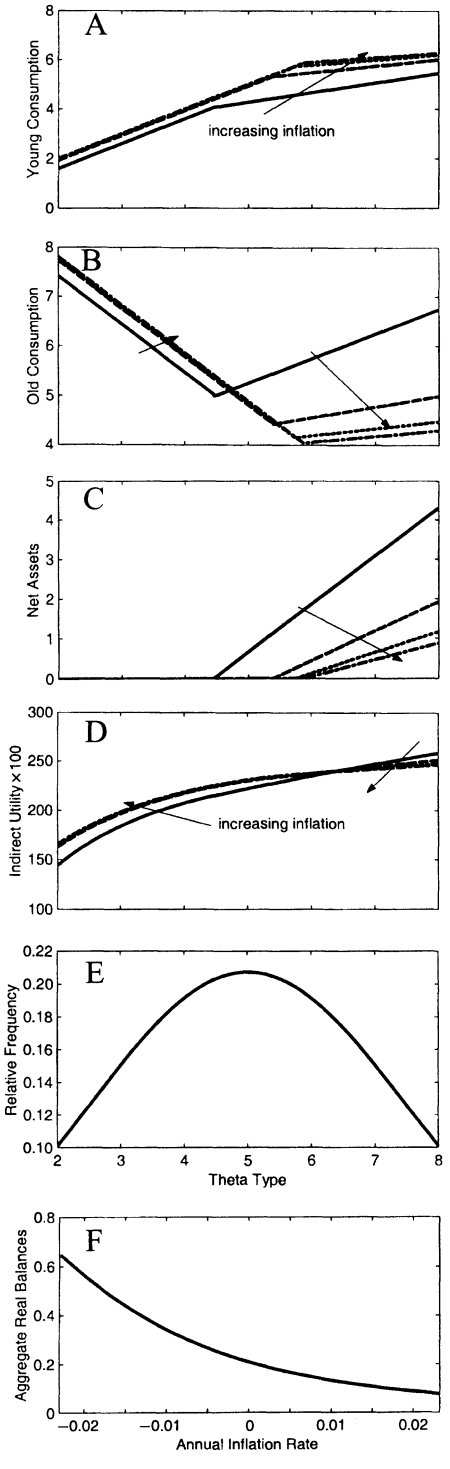

FIGURE 2. EFFECTS OF INCREASING INFLATION ON THE CROSS-SECTION DISTRIBUTION OF CONSUMPTION AND ASSETS HOLDINGS

assets, and utility across households of different θ types. To construct this figure, I set, as a simple benchmark, $\alpha = 1$, $\mathrm{CV}(\theta) = 0.31$, and $\vartheta = 5$ (zero skew), and then computed the distributions for a selection of different money growth rates, including $\mu = \beta$ and $\mu = 1$.

An inflationary policy raises the relative price of old-age consumption, causing households of all types to consume more in their youth. Notice that high-θ types are much more aggressive in their substitution toward young consumption. For relatively high-θ households, an inflationary policy means less old-age consumption (since they will save less), but for relatively low-θ households, the lump-sum transfers financed from inflationary revenues make more old-age consumption possible. An inflationary policy acts to flatten the distribution of household utility outcomes (as shown in Fig. 2D). But an inflationary policy comes at a cost: as steady-state inflation is increased, real balance holdings fall. Furthermore, the cutoff $\hat{\theta}$ increases, and a smaller fraction of households are net savers. Higher inflation makes self-insurance more expensive and leads to a shrinking tax base. These offsetting effects tend to ensure that an optimal policy will involve less-than-complete redistribution.

IV. Summary and Conclusions

In this paper I have provided an example of an economy where households can hold a portfolio of money and interest-paying bonds and have standard money demands, but where an inflationary policy can be the socially optimal compromise between self-insurance and social insurance. A deflationary policy, like Friedman's Rule, raises the rate of return on money and makes it easier for households to smooth consumption, but it also imposes lump-sum taxes which are especially burdensome for constrained households. An inflationary policy raises the relative price of old-age consumption, encouraging all households to consume more in youth. For relatively rich households, an inflationary policy means less old-age consumption (since they will save less), but for relatively poor households, the lump-sum transfer financed from inflationary revenues increases old-age consumption.

An inflationary policy tends to flatten the distribution of household utility outcomes, but

there is an optimal amount of redistribution beyond which the increasing costs of self-insurance and a diminishing tax base make any more inflation detrimental. An optimal policy has to balance these costs and benefits.

In the numerical examples presented above, an optimal policy can involve using inflation to redistribute resources in favor of would-be borrowers. Among other things, the extent of the optimal redistribution depends on the shape of the distribution of endowment income and the degree to which government finances are monetized.

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